Summary

In this meeting, I started by discussing the different types of PDE equations, then explained that the HJB equation is categorized as a nonlinear PDE. I mentioned that the viscosity solution is a weak solution for the HJB equation, and in the open-set toolboxes, the HJB equation is solved based on the viscosity solution. I also explained that I have no knowledge of the numerical method used in this toolbox.

Dr. Begum suggested that it is better not to focus on the numerical solution part since we are not experts in this area. Dr. Petrik also said, "Let's solve a specific example of the HJB equation." I replied that I already have the HJB equation for Dubin’s car, and I wrote it on the whiteboard, starting from the dynamics and formulating the LQR problem. I explained that during the last meeting, I was just trying to calculate the value function using this toolbox. It seems that the toolboxes are confusing when it comes to characterizing the equation being solved: Is it the HJB or just the HJI part of the HJB? How is the input being maximized, and, above all, how are the HJI or HJB equations connected to Nagumo’s theorem?

Dr. Yoon suggested that I derive Equation 16 from the overview paper, but honestly, it seems that the paper causes more confusion. Every time I try to find a connection between Nagumo’s theorem and HJIB, I get completely confused.

All the professors agreed that we should start with Nagumo’s theorem and then find a connection between them. I was considering learning about HJI instead of HJB—perhaps that would help me understand better, especially when disturbances are involved. It seems that I have all the components needed to put together to understand what the final equation is that we are trying to solve. However, Dr. Begum said the theorem is just discussing the dynamic system. I tried to convince them that it seems like the entire setup is for the HJI equation, from the signed distance function to the maximization part, as I referred to the general framework paper, and it seems that the lower bound of the HJI or HJB is the key to making the connections.

For me, Nagumo’s theorem in a safety context is very much like a Lyapunov stability function: if we have a function, or if we can find a good Lyapunov candidate function, then the derivative of that function with respect to time—if it is negative—means the system is moving towards zero or the origin. Similarly, Nagumo’s theorem states that if a function exists (since Nagumo’s theorem is about the existence of a solution curve), we can define a condition like the Lyapunov condition to ensure that the system remains in the invariant set.

Dr. Petrik also showed a differential games book, noting that it includes the HJI equation. He was also interested in the proof of Nagumo’s theorem, while Dr. Begum believes that understanding the theorem is more important. Dr. Yoon also mentioned that I must understand this intuitively.